VERMONT STATE MATHEMATICS COALITION TALENT SEARCH November 16, 2004

Test 2 of the 2004 - 2005 school year (Test 3 arrives at schools January 4, 2005)

Student Name _____ School _____

Grade _____ Math Department Head _____

Directions: Solve as many as you can of the problems and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for correct answers which are the result of guesses, conjectures or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. You may earn bonus points for "commendable solutions"- solutions that display creativity, ingenuity and clarity. Your answers and solutions must be postmarked by December 14, 2004 and submitted to Tony Trono, Vermont State Mathematics Coalition, 419 Colchester Avenue, Burlington, VT 05401. (For Coalition information and a copy of the test: http://www.state.vt.us/educ/vsmc)

1. Find four positive integers a, b, c, d that satisfy $a^2 + 2b^2 + 3c^2 = d^2$ for which $a + b + c + d$ is a minimum.

Answer: _____

2. How many three digit numbers can be formed for which the sum of the three digits is 17?

Answer: _____

3. The number x satisfies $3^{2x} - 34 \cdot 15^{x-1} + 5^{2x} = 0$. Calculate all possible values of $6 \cdot 3^{2x} + 78 \cdot 5^{2x}$.

Answer: _____

4. The area of a triangle ABC is k. AB is extended through B to point X such that AX = 3AB.
BC is extended through C to point Y such that BY = 3BC, and similarly
CA is extended through A to point Z such that CZ = 3CA.

Find the area of triangle XYZ in terms of k.

Answer: _____

5. The equation $ax^2 + bx + c = 0$ has two different roots including the root –2. The equation $ax^2 + cx + b = 0$ has two different roots including the root 3. Find the product of the four roots of the two equations.

Answer: _____

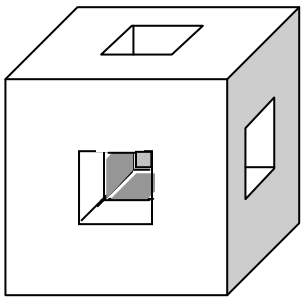
6. Solve for x if $\sqrt{x-1} - \sqrt{x-\sqrt{3x}} = \frac{\sqrt{x}}{2}$.

Answer: _____

7. For the positive integers a, b, and c, $a + b + c = 509$. The numbers $4a + 24$, $5b - 177$, and $3c + 148$ form an arithmetic progression. The three numbers $\frac{a}{2}, \frac{b+11}{5}$, and $\frac{c-52}{8}$ form a geometric progression. Find the number that is the sum of the three terms of the arithmetic progression.

Answer: _____

8. A cube with sides of length 6 inches has a square of sides x inches drawn on a side so that it is centered on the side. That square is the outline of a square hole drilled through the cube from one side through to the other side. Another square of side x inches is drawn on the top of the square, and outlines a square hole drilled through from the top to the bottom of the cube. Similarly, a square of sides x inches is centered on the front of the cube, and outlines a square hole drilled through from the front to the back of the cube.



a) Find x (to the nearest tenth of an inch) if 50% of the wood has been removed from the original cube.

b) Find x (to the nearest tenth of an inch) if 80% of the wood has been removed from the original cube.

Answer: a) _____ b) _____