VERMONT STATE MATHEMATICS COALITION TALENT SEARCH October 5, 2004

Test 1 of the 2004 - 2005 school year (Test 2 arrives at schools November 16, 2004)

Student Name _____ School _____

Grade _____ Math Department Head _____

Directions: Solve as many as you can of the problems and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for correct answers which are the result of guesses, conjectures or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. You may earn bonus points for "commendable solutions"- solutions that display creativity, ingenuity and clarity. Your answers and solutions must be postmarked by November 2, 2004 and submitted to Tony Trono, Vermont State Mathematics Coalition, 419 Colchester Avenue, Burlington, VT 05401. (For Coalition information and a copy of the test: http://www.state.vt.us/educ/vsmc)

1. The nine numbers in a magic square are consecutive odd numbers. Two numbers in the magic square are given.
N(1, 1) = 47, and N(3, 2) = 41.

Enter the nine numbers so that their sum is as large as possible.

47		
	41	

2. When the lengths of the three altitudes of a triangle are added in pairs, the resulting sums are 81, 96, and 105. Find the exact measurement of the smallest angle of the triangle.

Answer: _____

3. For the integers a, b, and c, then $\frac{2a-b}{c} = \frac{2b+c}{a} = \frac{-2a-c}{b}$.

If $a + b = 2004$, evaluate $a + b + c$.

Answer: _____

4. Find all possible sets of seven non-zero integers whose sum is equal to their product.

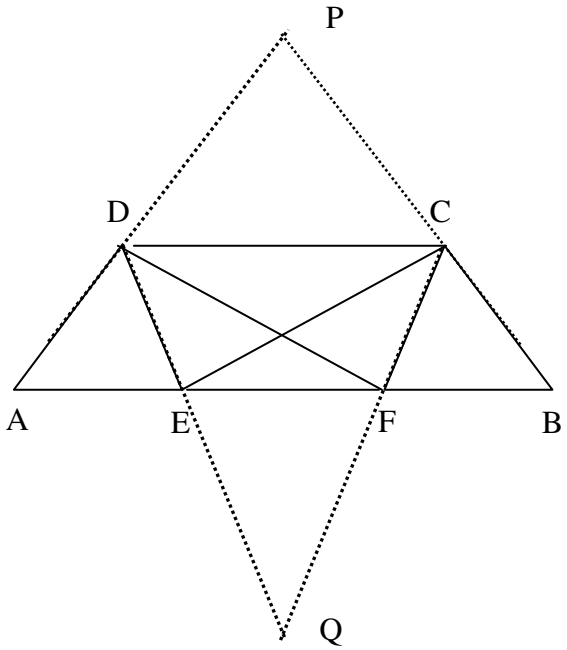
Answers: _____

_____

5. ABCD is a trapezoid with AB = 21 and CD = 13. You are given that $\Delta ADF \cong \Delta BCE \cong \Delta DEC \cong \Delta CFD$ and $AD \perp DF$.

a) Find the area of ABCD.

b) AD and BC are extended to meet at point P. DE and CF are extended to meet at point Q. By how much does the area of ΔDCP exceed the area of ΔEFQ ?



Answer: _____

6. The lengths of the sides of a triangle are 45, 60, and 75. A *median* of a triangle is a line segment from a vertex that bisects the opposite side. Point G is the center of gravity (the centroid) of the triangle. It is the point of intersection of the three medians. Find the sum of the three distances from G to the sides of the triangle.

Answer: _____

7. You are given the following system of equations.

$$\begin{aligned} x + ay + 3z + 2aw &= 9 \\ ax + 3y + 2az + w &= 7 \\ 3x + 2ay + z + aw &= 1 \\ 2ax + y + az + 3w &= 3 \end{aligned}$$

Given that $x + y + z + w = a$, find

a) all possible values of a.

b) (x, y, z, w) where x, y, z, w are rational numbers, but none of them are integers.

Answer: a)_____, b)_____

8. Integers a and k (where k is not 0) have been selected so that the parabola of the form $y = ax^2 + bx + c$ passes through the points (0, 0), (3k, 0), and (3k – 2, 20). Let S be the sum of the coordinates of the vertex point. Determine the smallest value of S.

Answer: _____