**Program 2015 Governor's Institute in Mathematical Sciences June 21 - 26, 2015**

Descriptions of the class sessions for this year's Institute:

**Sizes of Infinity**

How many letters are there in the alphabet? How many points are there on a line? The answer to the first question is the number 26. You might think the only answer we can give to the second question is "infinitely many." But it turns out that infinity comes in different sizes, and there are "numbers" that measure these different sizes of infinity. I will explain what this means and why it is true. We will also discuss the arithmetic of infinite numbers. This class is taught by Dan Velleman of the Amherst College Mathematics Department.

**Versatile Coins**

When an ordinary coin is flipped, it comes up heads with probability ½ and tails with probability ½. This can be useful if you need to make a random choice between two options. But what if you need to make a more complicated random choice? In that case, we'll see that coins with different probabilities of heads and tails can be useful. We'll see that carefully designed coins can do remarkable things For example, we'll find out why you might want to have a coin that comes up heads with probability $\left(5 + \sqrt{10\sqrt{15} - 25}\right)/10$. This class is taught by Dan Velleman of the Amherst College Mathematics Department.

**The Prosecutor's Fallacy, Bayes' Rule, and False-Positives**

Conditional probabilities and Bayes' Rule have useful applications in disease screening, jury deliberations, and everyday life. While the calculations can be tricky, communicating the results successfully can be just as tricky. We will look at several real-life examples, and consider how physicians and legal experts could best present probabilities. This class is taught by Sheila Weaver of the UVM Department of Mathematics & Statistics.

**Framing the Proof of the Pythagorean Theorem and Investigating Pythagorean Triple Properties**

We will begin this session with some hands-on proofs of the Pythagorean using sets of congruent right triangles and other famous methods, with historical connections to some ancient mathematicians and civilizations. We will then discuss Pythagorean triples and some of their properties, including some neat connections that they have with Fibonacci numbers. This class is taught by George Ashline of the St. Michael's College Mathematics Department.

**Slide Rules**

Slide rules have been around for centuries. In its most basic form, the slide rule uses two logarithmic scales to allow rapid multiplication and division of numbers. Combined with some nifty mental estimation shortcuts that were (somewhat) commonly used in the slide-rule age, as calculators and computers were rarely accessible. We explore some of these tricks that gave folks a better feel for numbers…out of necessity! This class is taught by Bill Gottesman.

**Hexaflexagons**

Flexagons are paper polygons, folded from straight or crooked strips of paper. They are folded such that one may "flex" them so as to allow their faces to change. Invented by a curious mathematics student,
Arthur Stone, they provide more than an idle curiosity. We will create some flexagons and explore some of their mathematical properties. This class is taught by John R. Schmitt of the Middlebury College Mathematics Department.

**The Tower of Hanoi**

Suppose that a positive integer number of discs of radii 1,2,…,n are stacked one on top of another in location A so that the disc of radius r is on top of the disc of radius r+1 for all r between 1 and n-1. We wish to move the entire stack, one disc at a time, to another location, which we¹ll call location B, but we have only one other location (location C) to stack discs and at all times we must observe the rule that a "large" radius disc may never be on top of a "small" radius disc. What's the minimum number of moves necessary to move the stack? This class is taught by John R. Schmitt of the Middlebury College Mathematics Department.

**Measuring Water**

Say that you have a 3-gallon jug, a 5-gallon jug, access to a large fountain of water and wish to measure out 4 gallons of water. You may: (1) fill a jug completely with water from a fountain, (2) empty a jug by pouring its contents into the fountain, and (3) pour water from one jug to the other until either the first jug is empty or the second one is full. If it is possible to do, how many moves will it take? This class is taught by John R. Schmitt of the Middlebury College Mathematics Department.

**Origami**

Who hasn't seen a paper crane and wonder how that was created? The practice and study of origami also encapsulates several subjects of mathematical interest. For instance, the problem of flat-foldability (whether a crease pattern can be folded into a 2-dimensional model) has been a topic of considerable mathematical study. A number of technological advances have come from insights obtained through paper folding. For example, techniques have been developed for the deployment of car airbags and stent implants from a folded position. This class is taught by Martin Chandler of UVM.

**Some surprising things you can do with triangles.**

We all know that the sum of the angles in a triangle in the plane is 180 degrees. In this talk we will see that triangles on spheres are much more interesting; different triangles have different angle sums. We will investigate this phenomenon and use what we discover to show a surprising result about triangulations of spheres. This class is taught by Emily Proctor of the Middlebury College Mathematics Department.

**3D Printing:**

When they needed a wrench on the international space station, instead of waiting for the next supply flight, they beamed up the design and used a 3D printer to make one on the spot. We'll talk about how 3-D printers work, watch a printer I made printing something, and pass around some things I've printed with it. We'll also talk about how you can design objects to be printed by adding, subtracting, intersecting, rotating, moving, and scaling a few basic 3D geometric figures. This class is taught by David Hathaway of the University of Vermont Computer Science Department and retired IBM engineer.

**Optimal Math**

We all want to get somewhere faster, use less of a resource to build something, complete a job in less time, and in general do more with less. We'll explore a few ideas from calculus that inform us about how to be a little more efficient in our everyday lives. Examples include route shortening/planning, materials minimization and other sundry topics. This class is taught by Greg Petrics of the Johnson State College Mathematics Department.

**Modular arithmetic and round robin tournaments**

In this lecture we will develop the idea of modular arithmetic. This is a very useful area a when studying things that are cyclic in nature such as days of the week or months of the year. We will learn how to do arithmetic (add, subtract multiply, divide, exponentiate) in modular systems. As an application, we will discuss how to easily construct a round robin tournament for any number of players (try this without modular arithmetic and you will see that this is a very hard thing to do, in general). This class is taught by Jeff Dinitz of the UVM Department of Mathematics & Statistics.

**Latin squares and the Euler 36 officer problem**

In 1782, the famous Swiss mathematician Leonhard Euler asked the following question: is it possible to arrange six regiments consisting of six officers, each of different ranks, in a $6 \times 6$ square so that no rank or regiment will be repeated in any row or column. We will show how this relates to so-called *Latin squares* (and even to Sudoku squares) and we will provide a solution to this problem. This will actually provide us with a second nice application of modular arithmetic. This class is taught by Jeff Dinitz of the UVM Department of Mathematics & Statistics.

**Graph theory: Planar graphs and platonic solids**

A graph is composed of a set of vertices (or nodes) and a set of edges that connect the vertices. Graphs are useful mathematical models for transportation, communication, social or supply networks, but are also very interesting in their own right. We will discuss some well-known graphs such as cycles, paths, complete graphs and the Petersen graph. A graph is said have an embedding on the plane, if it can be drawn without any of the edges crossing at points other than vertices. If a graph has such an embedding, it is called *planar*. We will determine which graphs are planar, as well as many that are not. Knowing which graphs are planar will help us discover the Platonic solids, as they are very special example of planar graphs. If time permits, we will discuss what it means and looks like to embed a graph on the torus or other higher surfaces. This class is taught by Jeff Dinitz of the UVM Department of Mathematics & Statistics.

**UVM AERO**

The University of Vermont Alternative Energy Racing Organization is a student-run group that works for the advocacy, research, and development of alternative energy drive systems. AERO models itself as a progressive engineering firm with primary goals to design, build, test, and race a highly competitive vehicle in the annual International Formula-Hybrid competition sponsored by IEEE and SAE. AERO is an opportunity to demonstrate the efficacy of non-traditional drive-systems for mobility engineering, and to drive fast at the same time! We will take a closer look at this organization, as well as tour their shop and see their race car. This session is led by Nathanial Rex of UVM AERO.

**Evolutionary Robotics**

Building a robot is hard work: it requires an understanding of physics, biology, math, mechanical and electrical engineering, and computer science. So rather than build a robot by hand, could we get a computer to build a robot for us? In this course, we explore how we can use computer programs known as `evolutionary algorithms' to create and compete virtual robots against one another, much like in Will Wright's 'Spore' computer game. Along the way, we'll learn much about biological evolution, computer simulation, and robotics. This class is taught by Josh Bongard of the UVM Computer Science Department.

**Problem Solving Strategies**

This session is taught by Matt Wu who is attending MIT this fall and a GIV Math Alumnus.