

Test 1 of the 2006 - 2007 school year

(Test 2 arrives at schools November 21, 2006)

Student Name _____

School _____

Grade _____

Math Department Head _____

Directions: Solve as many as you can of the problems and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for correct answers which are the result of guesses, conjectures or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. You may earn bonus points for "commendable solutions"- solutions that display creativity, ingenuity and clarity. Your answers and solutions must be postmarked by November 7, 2006 and submitted to Tony Trono, Vermont State Mathematics Coalition, 419 Colchester Avenue, Burlington, VT 05401. (For Coalition information and a copy of the test: <http://www.state.vt.us/educ/vsmc>)

1. Solve for n if $(10^{12} + 25)^2 - (10^{12} - 25)^2 = 10^n$

Answer: _____

2. We will be looking at paths from Box 1 to Box 8, with a move from one box to another made by certain rules. The permitted rules of moving take you from the number n to the next higher integer $n + 1$ (as from 3 to 4 or from 6 to 7), or from the number n to the number $n + 2$ (as from 1 to 3 or from 6 to 8).

1	3	5	7
2	4	6	8

All paths start at 1 and end at 8. Several examples of permitted paths are these (1, 3, 5, 7, 8), (1, 2, 3, 5, 6, 8), and (1, 2, 3, 4, 5, 6, 7, 8). Several paths not permitted are (1, 4, 6, 8), (1, 5, 7, 8), and (1, 2, 3, 4, 7, 8).

How many different permitted paths are there?

Answer: _____

3. After Heidi's birthday party, those on the clean-up crew were talking about their ages. All of them were teen-agers, and the product of their ages was 10,584,000.

a) How many were on the clean-up crew.

b) What was the sum of their ages?

Answers: a) _____ b) _____

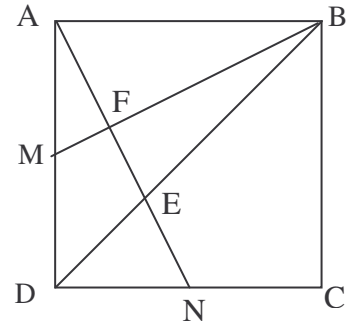
4. The function f is defined for the whole numbers 0, 1, 2, 3, 4,

$$f(n) = \begin{cases} k & \text{for } n = 0 \\ f(n-1) + 20n & \text{for } n > 0 \end{cases}$$

For what value of k will $f(7) = 2006$?

Answer: _____

5. The area of the square ABCD is 900 square units. M is the midpoint of side AD, and N is the midpoint of side DC. Find the area of the quadrilateral DEFM.



Answer: _____

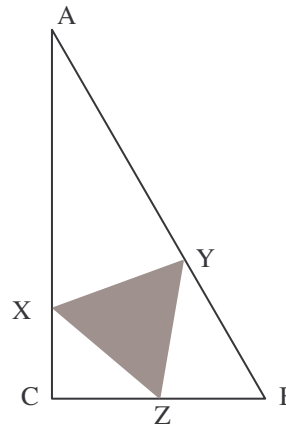
6. In the following multiplication problem, each letter represents a different digit. When the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 (in this order) are replaced by their letters, then the letters will spell out a phrase. What is the phrase?

$$\begin{array}{r}
 \text{USP} \\
 \times \text{RAP} \\
 \hline
 \text{OHEP} \\
 \text{SSTS} \\
 \hline
 \text{OUNR} \\
 \hline
 \text{OEEOAP}
 \end{array}$$

Answer: _____

7. You are given that $\triangle ABC$ is a right triangle and $\angle A$ is 30 degrees. The $\triangle XYZ$ is an equilateral triangle, and $CZ = BZ = 1$.

Find the area of $\triangle XYZ$.



Answer: _____

8. Solve for x if $\frac{6}{\sqrt{x-8}-9} + \frac{1}{\sqrt{x-8}-4} + \frac{7}{\sqrt{x-8}+4} + \frac{12}{\sqrt{x-8}+9} = 0$.

Answer: _____