Vermont State Mathematics Coalition Talent Search   November 17, 2010

Test 2 of the 2010 – 2011 school year

PRINT NAME:_____ Signature:_____
Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School_____ Grade_____

Directions: Solve as many of the problems as you can and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for inadequately supported correct answers and/or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. Solutions that display creativity, ingenuity and clarity may receive special recognition and commendation. Your solutions must be postmarked by December 08, 2010 and submitted to:

Problem 1.
An ellipse *E* has two circles, *C*1 and *C*2, inscribed in it, such that *C*1's center is the center of *E* and *C*2's center is one of the two foci of *E*. If *C*1 is a unit circle and *C*1 and *C*2 are externally tangent, find the area of *E*.

Answer: _____

Problem 2.
Find the remainder when $f(x) = x^{57} + x^{19} + x^9 + 2010$ is divided by $x^3 - x$.

Answer: _____

Problem 3.
In acute triangle ABC, altitudes AY and BX are drawn to sides BC and AC respectively. If $BY = 20$ and $AY = 99$, find the sine of angle AXY.

Answer: _____

Problem 4.
The first, second and third terms of an arithmetic progression (AP) are $x^2 - 6x + 4$, $3x^2 - 11x + 2$, and $2x^2 - x - 12$ respectively.
The nth term of the progression equals -2011. Find *n*.

Answer: _____

Problem 5.
It's well known that country music songs often emphasize love, prisons and trucks. A recent survey found the following:

       a. 12 songs were about a truck driver who was in love while in prison.
       b. 13 songs were about a prisoner in love.
       c. 18 were about truck driver in love.
       d. 28 were about a person in love.
       e. 3 were about a truck driver in prison, but not in love.
       f. 16 were about truck drivers who were not in prison.
       g. 8 were about a person out of prison, who is not in love, and didn't drive a truck.
       h. 2 songs were about a prisoner who was not in love and didn't drive a truck.

Find   a) the number of songs in the survey   _____
          b) the number of songs about
                1. truck drivers   _____
                2. prisoners   _____
                3. truck drivers in prison   _____
                4. people not in prison   _____
                5. people not in love   _____

Problem 6.
Three distinct integers are randomly chosen from the set $S = \{1, 2, 3...10\}$. What is the probability that their product is a perfect cube.

Answer: _____

Problem 7.
The equation $x^3 + ax^2 + bx + c = 0$ has real non-zero roots. If two of the roots are addition inverses and two are multiplicative inverses, determine the maximum possible value of $(b + c)$.

Answer: _____

Problem 8.
A positive integer *N* leaves a remainder of 1 when divided by 3, a remainder of 3 when divided by 5, a remainder of 5 when divided by 7, a remainder of 7 when divided by 9 and a remainder of 10 when divided by 11. Find the smallest such *N* .

Answer: _____

Note: Test 3 will be available at
http://www.vtmathcoalition.org/talent-search/
on February 2, 2011.

To receive the next tests via email, clearly print your email address below:

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