Vermont State Mathematics Coalition Talent Search January 5, 2015
Test 3 of the 2014 – 2015 school year

PRINT NAME:_________________________ Signature:_____________________
Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School____ Grade____________

Current Mathematics Teacher: ____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final." Your solutions may be emailed to johlson@sbschools.net or be postmarked by January 30, 2015 and submitted to:

Jean Ohlson
Vermont State Math Coalition
PO Box 384
Charlotte, VT 05445

To receive the next tests via email, clearly print your email address below:

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The Vermont Math Coalition's Talent Search test is prepared by Jean Ohlson (Math Teacher at South Burlington HS) and Evan Dummit (Visiting Assistant Professor of Mathematics at University of Rochester)

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Problem 1.

At the Mathville Tapas restaurant, the dishes come in three types: small, medium, and large. Each dish costs an integer number of dollars, with the small dishes being the cheapest and the large dishes being the most expensive. (Tax is already included, different sizes have different prices, and the prices have stayed constant for years.) This week, Jean, Evan, and Katie order 9 small dishes, 6 medium dishes, and 8 large dishes. When the bill arrives, the following conversation occurs:

Jean: "The bill is exactly twice as much as last week."

Evan: "The bill is exactly three times as much as last month."

Katie: "If we gave the waiter a 10% tip, the total would still be less than $100."

Find the price of the group's meal next week: 2 small dishes, 9 medium dishes, and 11 large dishes.

Answer: ____________

Problem 2.

Points A, B, C, and E lie on circle O such that AC is a diameter of O. Point D lies outside the circle on the perpendicular bisector of CE such that angle $DEA = 150^o$. If $\angle EAB = 75^o$ and $AB = BC = 4$, find the perimeter of pentagon $ABCDE$.

Answer: ____________

Problem 3.

Let $[x]$ be the greatest integer function and $\{x\} = x - [x]$ be the fractional part of x, (For example, $[\pi] = 3 \; and \; \{\pi\} = \pi - 3$). Find the number of real numbers x in the interval [0,1] such that $\{x\} + \{2x\} + \{3x\} + \{4x\} + \{5x\} = 2$.

Answer: ____________

Problem 4.

Four boys and four girls each bring one gift to a party. Each boy randomly chooses a girl to give his gift to, and each girl randomly chooses a boy to give her gift to. Determine the probability that each person receives exactly one gift and that no two people exchanged gifts directly with one another (i.e., if B gave G a gift, then G did not give B a gift).

Answer: __________

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Problem 5.

A set of n distinct positive integers has sum 2015. If every integer in the set has the same sum of digits (in base 10), find the largest possible value of n.

Answer: __________

Problem 6.

A checkerboard is "almost tileable" if there exists some way of placing non-overlapping dominoes on the board that leaves exactly one square in each row and column uncovered. (Note that dominoes are $2x1$ tiles which may be placed in either orientation.) Prove that, for $n \geq 3$, an nxn checkerboard is almost tileable if and only if n is congruent to 0 or 1 modulo 4.

Note: For this problem, please include your proof on a separate sheet of paper.

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