Vermont State Mathematics Coalition Talent Search October 24, 2016

Test 2 of the 2016 – 2017 school year

PRINT NAME:__________________________ Signature:______________________

Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School____ Grade____________

Current Mathematics Teacher: ____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets**, in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final." Your solutions may be emailed to johlson@sbschools.net or be postmarked by **November 18, 2016** and submitted to:

Jean Ohlson
Vermont State Math Coalition
PO Box 384
Charlotte, VT 05445

To receive the next tests via email, clearly print your email address below:

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The Vermont Math Coalition's Talent Search test is prepared by Jean Ohlson (Math Teacher at South Burlington HS) and Evan Dummit (Visiting Assistant Professor of Mathematics at University of Rochester)

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1) Three students on Vermont's team are at the American Regional Math League Competition are standing in a large open field. Matthew and Kailey are standing in the same spot and Kevin is standing 12 meters away. Matthew chooses a random direction and walks 12 meters in this direction. What is the probability that Matthew is closer to Kevin than Kailey is to Kevin?

Answer: ____________

2) Erik has a rectangular piece of cardboard measuring *x* units by y units. He cuts out four identically-oriented rectangles (with sides parallel to the full piece of cardboard) of dimensions 2 units by 4 units from the four corners of his piece of cardboard, in such a way that the resulting material can be folded into a rectangular box (including the top) whose volume is 72 cubic units. Find the area of Erik's original piece of cardboard.

Answer: ____________

3) The positive integer N is divisible by every two-digit integer. If N is written as the product of k two-digit integers, find the smallest possible value of k.

Answer: _____________

4) Let $a = \log_3 5$, $b = 2\log_2 a$, $c = 9^b$, $d = \log_a\left(1/c\right)$, $e = 3ad$, $f = 5^{4/e}$ and $g = \log_{\sqrt{2}}\left(2/f^3\right)$. Compute the value of *g*, in simplest form.

Answer: ____________

5) For positive real numbers x, y, and z, let $f(x,y,z) = min\left(\frac{2}{x}, \frac{2z}{y}, x^2 + 2xy, \frac{x}{z}\right)$. Find with proof, the maximum value of $f(x,y,z)$ and all triples (x,y,z) at which this maximum is achieved.

Note: For this problem, please include your proof on a separate sheet of paper.

6) In triangle ABC, the point *D* on *AB* and the point *E* on *AC* are chosen such that a circle of radius 8 can be inscribed in quadrilateral $BCED$ and a circle of radius 3 can be inscribed in triangle ADE. If $BC = 26$ and $BD + CE = 36$, find the perimeter of ΔABC.

Answer: ____________

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