Vermont State Mathematics Coalition Talent Search -- September 2021
Test 1 of the 2021-2022 school year

PRINT NAME:_____ Signature:_____
Note: Your signature indicates that answers provided herein are your own work and you have not asked for or received aid in completing this Test.

School_____ Grade _____

Current Mathematics Teacher: _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets,** in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final. Your solutions may be e-mailed to kmaccormick@cvsdvt.org or be postmarked by **October 8, 2021** and submitted to

Kiran MacCormick
Champlain Valley Union High School
369 CVU Road
Hinesburg, VT 05461

To receive the next tests via email, clearly print your email address below:

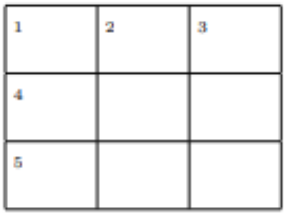
_____

The Vermont Math Coalition's Talent Search test is prepared by Kiran MacCormick (Math Teacher at Champlain Valley Union HS) and Evan Dummit (Assistant Teaching Professor at Northeastern University).

Vermont State Mathematics Coalition Talent Search -- September 2021
Test 1 of the 2021-2022 school year

1. In the cross-number puzzle below, each entry is a digit from 1-9. Solve the puzzle.

Across:	Down:
1. A 3-digit multiple of 37.	1. The same number as 3-down.
4. An even perfect square.	2. A multiple of 47 less than 200.
5. Twice a perfect cube.	3. The same number as 1-down.



2. This is a relay problem. The answer to each part will be used in the next part.
   (a) Suppose A and B are distinct, nonzero digits such that $2 \cdot \underline{ABA} + 3 \cdot \underline{BB} + 1 \cdot \underline{A} = \underline{BBB}$ (where $\underline{ABA}$ means the 3-digit integer whose digits are $A, B, A,$ etc.) What is the value of the two-digit integer $\underline{AB}$?
   (b) Let T be the answer to part (a). What is the least positive integer that is a multiple of $T+3$ whose sum of digits is $T-3$?
   (c) Let S be the closest integer to the answer of part (b), divided by 120. A rectangle is inscribed in the circle $x^2 + y^2 = S$ and its side lengths are both integers. What is the area of this rectangle?

Answers: (a) _____ (b) _____ (c) _____

3. Ragulan has a rectangular piece of cardboard measuring x units by y units. He is interested in cutting out four squares of equal side length, one from each corner of the rectangle, to form a new piece of cardboard which he may then fold up into a rectangular box missing its top. If he cuts out a square of side length 2 units from each corner, then the resulting box has volume 72 cubic units. If instead he cuts out squares of side length 3 units, then the box has volume 42 cubic units. Find the area of the original piece of cardboard.

Answer:_____

4. Suppose that x is a real number such that $2^{4^{8^x}} = 4^{8^{2^x}}$. What is the value of $4^x - 2^x$?

Answer:_____

5. Karen writes down the values of the products $a \times b$ where a and b are integers with $1,000,000,000,000,000,000 \leq a \leq 1,000,000,002,000,000,001$ and $1,000,000,000,000,000,000 \leq b \leq 1,000,000,002,000,000,001$, and then she erases any duplicate values. After erasing, how many different numbers does Karen have written down?

Answer:_____

6. A positive integer is called *power-different* if it can be written as the difference between a power of 2 and a power of 3. For example, $1 = 3 - 2, 2 = 3 - 1,$ and $15 = 16 - 1$ are all power-different. Determine, with proof, the least prime number that is not power-different.

Note: For this problem, please include your proof on separate sheets of paper.

The Vermont Math Coalition's Talent Search test is prepared by Kiran MacCormick (Math Teacher at Champlain Valley Union HS) and Evan Dummit (Assistant Teaching Professor at Northeastern University).