Vermont State Mathematics Coalition Talent Search -- October 2022
Test 1 of the 2022-2023 school year

PRINT NAME:_____ Signature:_____
Note: Your signature indicates that answers provided herein are your own work and you have not asked for or received aid in completing this Test.

School_____ Grade _____

Current Mathematics Teacher: _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets,** in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final. Your solutions may be e-mailed to kmaccormick@cvsdvt.org or be postmarked by **November 11, 2022** and submitted to

Kiran MacCormick
Champlain Valley Union High School
369 CVU Road
Hinesburg, VT 05461

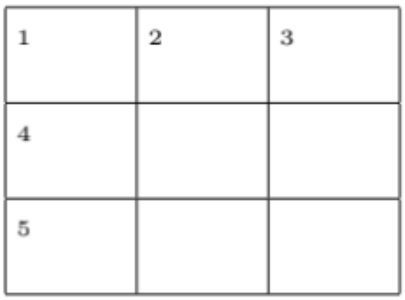
To receive the next tests via email, clearly print your email address below:

_____

1. In the cross-number puzzle below, each entry is a digit from 1-9. Solve the puzzle.

Across:
1. A power of 2.
4. A multiple of 11.
5. A multiple of 17.

Down:
1. A prime.
2. A perfect square.
3. A power of 5.



The Vermont Math Coalition's Talent Search test is prepared by Kiran MacCormick (Math Teacher at Champlain Valley Union HS) and Evan Dummit (Assistant Teaching Professor at Northeastern University).

2. This is a relay problem. The answer to each part will be used in the next part.
   (a) A set of positive integers is called "divisible" if each element in the set is divisible by the smallest element in the set. For example, the sets {2, 4, 16}, {1, 3, 30}, and {12} are all divisible. How many different nonempty subsets of {1,2,3,4,5,6,7,8,9,10} are divisible?
   (b) Let A be the answer to part (a). If a, b and c are all positive integer divisors of $A-1$, and $a+b=c$ where a is even and b is odd, what is the largest possible value of a?
   (c) Let B be the answer to part (b). A cone with vertex angle measuring $(B+12)^\circ$ is inscribed in a sphere of radius $B/9$. What is the volume of the cone?

Answers: (a) _____ (b) _____ (c) _____

3. Diaz has 54 fair 6-sided dice. One day, he stacks 27 of them in a 3 x 3 x 3 cube shape, with all of the dice in the same orientation, and then separates them again and mixes them with his other 27 standard dice. However, while stacked, the numbers on all of the interior faces of the dice have worn off, leaving them blank. (The numbers visible on the exterior of the 3 x 3 x 3 cube are unaffected.) Diaz picks a random die from his collection of 54 and rolls it 3 times, obtaining no blank faces. What is the probability that his die has at least one blank face?

Answer:_____

4. Let F_n denote the nth Fibonacci number, defined by $F_1=F_2=1$ and $F_n=F_{n-1}+F_{n-2}$ for $n\geq 2$. Quadrilateral $VMTS$ is inscribed in circle O, where $VM=MT=2F_{2022}$, $TS=2F_{2023}$, and $SV=2F_{2024}$. The area of quadrilateral $VMTS$ can be written in the form $F_aF_b\sqrt{F_c}$ where a, b, and c are positive integers. Find the value of $a+b+c$.

Answer:_____

5. Due to rampant inflation and a coin shortage, it is very difficult to make change in Kiranistan. Currently, there are only three types of bills available, and they are respectively worth 1935∃, 2021∃, and 2115∃. Evan has a large supply of each bill denomination, and must make exact payments in all his purchases due to government regulations.

   (a) Prove that Evan cannot make exact payment for a purchase valued at 175,819∃.
   (b) Prove that Evan can make exact payment for any purchase valued at least 175,820∃.

Note: For this problem, please include your proof on separate sheets of paper.

6. Find the number of polynomials $p(x)$ of degree 8, all of whose coefficients are positive nonzero digits 1-9 inclusive, such that $p(x)$ is divisible as a polynomial by $x^2-(i\sqrt{3})x-1$ where $i^2=-1$. (For example, one such polynomial is $p(x)=x^8+5x^7+5x^6+9x^5+7x^4+9x^3+6x^2+4x+2$.) Answer:_____

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