

Vermont State Mathematics Coalition Talent Search -- September 2019

Test 1 of the 2019-2020 school year

PRINT NAME: _____ Signature: _____

Note: Your signature indicates that answers provided herein are your own work and you have not asked for or received aid in completing this Test.

School _____ Grade _____

Current Mathematics Teacher: _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets**, in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final. Your solutions may be e-mailed to kmaccormick@fnwsu.org or be postmarked by **October 9, 2019** and submitted to

Kiran MacCormick
Missisquoi Valley Union High School
175 Thunderbird Drive
Swanton, VT 05488

To receive the next tests via email, clearly print your email address below:

1. Erika has a special calculator that can do any of the basic arithmetic operations (addition, subtraction, negation, multiplication, division, exponentiation, and square roots), but only has four digit buttons 1, 3, 4, and 6. Operations may be used multiple times, but each digit button must be pressed exactly once during a calculation. Four example calculations Erika could make are given below.

Input	Result
$4 \times 6 + 3 + 1$	28
$6 - (4 \times 3 + 1)$	-7
$\sqrt{4^6} - 3 \div 1$	61
$(4 + 1)^{6-3}$	125

Using her calculator, how many of the positive integers from 1 to 20 inclusive can Erika create?

Answer: _____

2. Under a court order, a certain company was mandated to produce 17 million pages of internal documents. Given that a ream of paper contains 500 pages, that a box of 10 reams of paper measures 12 inches by 18 inches by 10 inches, and that a standard shipping container measures 8 feet by 8.5 feet by 20 feet, what is the smallest number of whole standard shipping containers that would be needed to contain all of the documents?

Answer: _____

3. In the magic square below, all of the entries are prime numbers forming an arithmetic progression, and the largest value has been given. Fill in the remaining entries, given that the upper right entry is the second-largest prime.

2089		

4. Two sides of a triangle have lengths 12 and 15, and the area of the triangle is $10\sqrt{65}$. Find all possible values for the length of the remaining side of the triangle.

Answer: _____

5. A robot is placed on the portion of a grid, pictured below, and programmed to move one unit each minute randomly in one of the four possible directions parallel to the grid (i.e., up, down, left, or right). It starts at the square marked "S", and if it moves into one of the squares marked "x", the robot falls off the grid. What is the probability that the robot will enter the square marked "E" at least once before it falls off the grid?

x	x	x	x	x
x			E	x
x		x		x
x	S			x
x	x	x	x	x

Answer: _____

6. Four distinct points are chosen in the xy -plane.

- If all four points lie on the parabola $y = x^2$, prove that there is a circle passing through the four points if and only if the sum of their x -coordinates is 0.
- If all four points lie on the hyperbola $y = 2/x$, prove that there is a circle passing through the four points if and only if the product of their y -coordinates is 4.

Note: For this problem, please include your proof on separate sheets of paper.