Vermont State Mathematics Coalition Talent Search -- October 2019
Test 2 of the 2019-2020 school year

PRINT NAME:_____ Signature:_____
Note: Your signature indicates that answers provided herein are your own work and you have not asked for or received aid in completing this Test.

School_____ Grade _____

Current Mathematics Teacher: _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets,** in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final. Your solutions may be e-mailed to kmaccormick@fnwsu.org or be postmarked by **November 22, 2019** and submitted to

Kiran MacCormick
Missisquoi Valley Union High School
175 Thunderbird Drive
Swanton, VT 05488

To receive the next tests via email, clearly print your email address below:

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The Vermont Math Coalition's Talent Search test is prepared by Kiran MacCormick (Math Teacher at Missisquoi Valley Union HS) and Evan Dummit (Assistant Teaching Professor at Northeastern University).

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1. A cryptarithm is a correct arithmetic equation encoded using letters such that each appearance of a letter has the same value (a digit 0-9) and such that different letters have different values. (Note that numbers are not allowed to have leading zeroes.) The cryptarithm given below has seven possible solutions. If O = 7 and R = 5, what is the value of the word OUTWEIGH?

```
   TWO
   TWO
+ FOUR
------
 EIGHT
```

Answer:_____

2. Twenty couples attend a party. During the party, each pair of male partygoers chats twice, and each pair of female partygoers chats once. If a total of 822 chats occurred during the party, what is the greatest possible number of male-female couples who could have attended the party in total?

Answer:_____

3. Find all ordered pairs (x, y) of nonzero rational numbers satisfying the equations $x^{y-3x} = y^5$ and $y^{y-3x} = x^{20}$.

Answer:_____

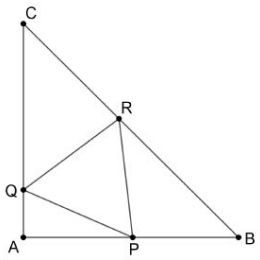
4. Suppose that A, B, C, and D are distinct lattice points in the plane such that all six lengths AB, AC, AD, BC, BD, and CD are integers. Prove that the product of these six lengths is divisible by 12.

Note: For this problem, please include your proof on separate sheets of paper.

5. For a positive integer n, let $f(n) = \sum_{d|n} i^d$, where $i = \sqrt{-1}$. Thus, for example, $f(10) = i^1 + i^2 + i^5 + i^{10} = -2 + 2i$. Find the smallest positive integer n for which $f(n) = 20 + 20i$.

Answer:_____

6. Right triangle ABC has side lengths 7, $4\sqrt{3}$, and $\sqrt{97}$. Equilateral triangle PQR is contained in triangle ABC, with one vertex on each side, as shown below. Find the minimum and maximum possible areas of triangle PQR.



Maximum area:_____

Minimum area:_____

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