Vermont State Mathematics Coalition Talent Search -- September 2023
Test 1 of the 2023-2024 school year

PRINT NAME:_______________________________________ Signature:_________________
Note: Your signature indicates that answers provided herein are your own work and you have not asked for or received aid in completing this Test.

School____ Grade _____________

Current Mathematics Teacher: ____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets,** in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final. Your solutions may be e-mailed to kmaccormick@cvsdvt.org or be postmarked by **October 18, 2023** and submitted to

Kiran MacCormick
Champlain Valley Union High School
369 CVU Road
Hinesburg, VT 05461

To receive the next tests via email, clearly print your email address below:

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1. Solve the cross-number puzzle below, where each entry is a digit from 0-9 and no answer starts with a 0..

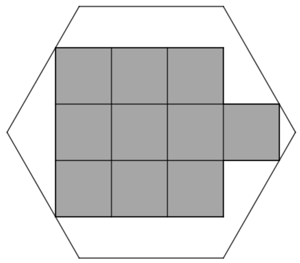
Across:
1. A perfect cube.
4. A divisor of 7!.
5. A sum of two 9th powers.

Down:
1. A divisor of 7!.
2. A perfect square.
3. A divisor of 9999.

1	2	3
4		
5		

The Vermont Math Coalition's Talent Search test is prepared by Kiran MacCormick (Math Teacher at Champlain Valley Union HS) and Evan Dummit (Associate Teaching Professor at Northeastern University).

2. The shaded region inside the regular hexagon pictured below consists of ten identical squares. To the nearest tenth of a percent, what percent of the hexagon's area is shaded?



Answer:____________

3. This is a relay problem. The answer to each part will be used in the next part.
   (a) Find the largest value of n such that there exists a convex n-gon whose angle measures (in degrees) are all prime numbers, and the angle measures are not all equal.
   (b) Let A be the answer to part (a). Let K be the greatest integer less than $\sqrt{A}$. A right triangle has a circle of radius 2 inscribed in it, and the triangle is itself inscribed ina circle of radius K. Find the sum of the leg lengths of the triangle.
   (c) Let B be the answer to part (b). The sequence $a_1, a_2, a_3, \cdots$ of positive integers has the property that $a_{n+1} = a_n + 2a_{n-1} + 1$ for each $n \geq 2$. If $a_8 = B^2$, what is the value of $a_1 + a_2$?

Answers: (a) ___________________ (b) ______________________ (c) ____________________

4. Kiran has a sequence of 2023 increasingly unfair coins: his first coin has a probability 1 of landing heads, his second coin has a probability ½ of landing heads, his third coin has a probability ⅓ of landing heads, and so forth, and his 2023rd coin has a probability 1/2023 of landing heads. Kiran starts with $1 and then flips each of the coins once in succession. Each time a coin lands heads, he doubles his money, and each time a coin lands tails, his money is unchanged. What is the expected value of the amount of money Kiran has after flipping all 2023 coins?

Answer:____________

5. Evan has a set of 21 Fibonacci coins whose values are the Fibonacci numbers $F_2 = 1$, $F_3 = 2$, $F_4 = 3$, $F_5 = 5$, $F_6 = 8$, and so forth, up to $F_{22} = 17711$. Evan wishes to give some of the coins to Alta and the rest to Michael, so that the total value of Alta's coins equals the total value of Michael's coins. In how many different ways can this be done?

Answer:____________

6. Prove that there exists a positive integer N such that a 2023-dimensional unit cube can be dissected into exactly d smaller cubes for every integer $d \geq N$.

Note: For this problem, please include your proof on separate sheets of paper.

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