

Vermont State Mathematics Coalition Talent Search -- September 2024

Test 1 of the 2024-2025 school year

PRINT NAME: _____ Signature: _____

Note: Your signature indicates that answers provided herein are your own work and you have not asked for or received aid in completing this Test.

School _____ Grade _____

Email address: _____

Current Mathematics Teacher: _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets**, in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final. Your solutions may be e-mailed to kmaccormick@cvsdvt.org or be postmarked by **October 28, 2024** and submitted to

Kiran MacCormick
Champlain Valley Union High School
369 CVU Road
Hinesburg, VT 05461

DISCLAIMER: Please consider completing the following name and image release form in order to help the Vermont State Mathematics Coalition promote our programs and support our pursuit of creating in Vermont significant and lasting improvements in mathematics education and a wider appreciation of mathematics. Signing this release form will not affect the scoring of the competition. If you are under 18, this will need to be signed by a parent or guardian. You only have to complete the image release form once.

NAME AND IMAGE RELEASE FORM

I hereby grant the Vermont State Mathematics Coalition and its representatives the right to take photographs of me, and I authorize the Vermont State Mathematics Coalition to copyright, use and publish the same in print and/or electronically. I agree that the Vermont State Mathematics Coalition may use such photographs of me with or without my name for any lawful purpose, including for example such purposes as publicity and web content. I also consent to the use of my name in connection with the Vermont State Mathematics Coalition. I hereby release and discharge the Vermont State Mathematics Coalition and its affiliates from any and all claims of any kind in connection with the use of the images and/or my name.

I have read and understand the above:

Signature _____

Printed name _____

Address _____

Date _____

Signature, parent or guardian _____ (if under age 18)

Parent/guardian name (printed): _____

$$\frac{ABC - DEF}{GH - IJ}$$

1. Find the greatest possible value of the ratio $\frac{ABC - DEF}{GH - IJ}$ where $A, B, C, D, E, F, G, H, I, J$ are distinct digits.

Answer: _____

2. This is a relay problem. The answer to each part will be used in the next part.
- (a) Ariel has 50% more candy bars than Belle, who in turn has 50% more candy bars than Cinderella. Ariel then gives 20 candy bars to Belle and N candy bars to Cinderella: afterwards, all three have the same number of candy bars. What is the value of N ?
- (b) Let A be the answer to part (a). A regular polygon has an internal angle measuring $A + 10$ degrees. How many diagonals does this polygon have?
- (c) Let B be the answer to part (b). A total of B blue marbles and 27 red marbles are placed into a bag. Two marbles are drawn randomly without replacement. What is the probability that the marbles are the same color?

Answers: (a) _____ (b) _____ (c) _____

3. Solve the cross-number puzzle below, where each entry is a digit from 1-9 (there are no 0's):

1	2	3	4
5			
6			

Across

- A perfect cube.
- A product of five distinct primes that sum to 42.
- A multiple of 81 whose digits are in decreasing order.

Down

- A prime less than 200.
- An odd perfect square.
- A multiple of 11.
- A prime greater than 200.

4. Find the unique ordered triple (x, y, z) of real numbers such that

$$x^2 + y^2 + z^2 + xy + xz + yz + 1243 = 63x + 47y + 58z.$$

Answer: _____

5. Elias has a circular pizza. He randomly chooses 2025 pairs of points on the circumference of the pizza and makes a straight cut along the line joining each of his 2025 pairs of points. Find the expected number of pieces into which the pizza has been divided after Elias makes all 2025 cuts.

Answer: _____

6. Suppose that a and b are positive integers such that ab divides $a^2 + b^2 + 1$. Prove that both a and b must be Fibonacci numbers. (Recall that the Fibonacci numbers are defined by $F_1 = F_2 = 1$ and $F_{n+1} = F_n + F_{n-1}$ for each $n \geq 2$.)

Note: For this problem, please include your proof on separate sheets of paper.