Vermont State Mathematics Coalition Talent Search -- January 2025
Test 3 of the 2024-2025 school year

PRINT NAME:_____ Signature:_____
Note: Your signature indicates that answers provided herein are your own work and you have not asked for or received aid in completing this Test.

School_____ Grade _____

Email address:_____

Current Mathematics Teacher: _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets,** in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final. Your solutions may be e-mailed to kmaccormick@cvsdvt.org or be postmarked by **February 16, 2025** and submitted to

Kiran MacCormick
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369 CVU Road
Hinesburg, VT 05461

The Vermont Math Coalition's Talent Search test is prepared by Kiran MacCormick (Math Teacher at Champlain Valley Union HS), Evan Dummit (Associate Teaching Professor at Northeastern University), and Elias Leventhal (Student at Yale University and former talent search winner).

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1. This is a relay problem. The answer to each part will be used in the next part.
   (a) What is the smallest positive integer greater than 120 that has the same set of prime divisors as 120?

   (b) Let A be the answer to part (a). Evan writes the integers $1, 2, 3, \ldots, A-1$ on a blackboard. He circles two of these integers and multiplies them to get a product P. If P is also equal to the sum of the other $A-3$ integers that Evan didn't circle, what is the value of the smaller of the two circled integers?

   (c) Let B be the answer to part (b). Kiran buys a total of $B-3$ cases of batteries for his automatic grading machine, which uses 6 batteries at a time. Each case has 24 boxes in it, and each box has 12 batteries in it. In each box, ¼ of the batteries are defective and do not work at all, while the remaining batteries will provide power for 30 minutes each. For how many total weeks will Kiran be able to use his grading machine, assuming the batteries are in constant use?

Answers: (a) _____ (b) _____ (c) _____

2. In triangle ABC, there exist points D, E, F on side BC (in that order from B to C) and points G, H, I on side AC (in that order from A to C) such that $AB = AD = BG = CF = CI = DH = EI = EG = FH = 1$. Find the degree measure of $\angle ABC$.

Answer:_____

3. Checkers are placed on a 45 x 45 gameboard, with $1, 2, 3, \ldots, 45$ checkers in the squares in the top row (from left to right), $46, 47, \ldots, 90$ in the next row (left to right), and so forth, with $1981, 1982, \ldots, 2025$ checkers in the bottom row (left to right). Evan then performs a series of moves, each move consisting of adding or removing one checker from all squares in any $2 \times 3, 3 \times 2, \text{or } 4 \times 4$ rectangular region on the board. After a sequence of these moves, only one square has checkers remaining. Determine all possible values for the number of checkers remaining on that square?

Answer:_____

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4. For two quadratic polynomials $f_1(x) = a_1x^2 + b_1x + c_1$ and $f_2(x) = a_2x^2 + b_2x + c_2$, we define their *coefficient distance* to be $\max(|a_1 - a_2|, |b_1 - b_2|, |c_1 - c_2|)$. For example, the coefficient distance between $2x^2 + x - 3$ and $x^2 + 3x - 1$ is $\max(|2 - 1|, |1 - 3|, |-3 - (-1)|) = 2$. If S is the set of all quadratic polynomials whose roots are real numbers, find the minimum possible coefficient distance between $20x^2 + 25x + 52$ and a polynomial in S.

Answer:_____

5. Recall that the Fibonacci numbers $F_1, F_2, \cdots$ are defined via $F_1 = F_2 = 1$ and $F_{n+1} = F_n + F_{n-1}$ for each $n \geq 2$. Find an ordered triple (a, b, c) of integers with $|a|, |b|, |c| < 10,000,000$ such that
$F_1 + 4F_2 + 9F_3 + 16F_4 + ... + 2025^2F_{2025} = aF_{2027} + bF_{2026} + c.$

Answer:_____

6. If N is a positive integer, a *chop* of N is obtained by introducing one or more plus signs between the digits of N, and adding the results. For example, two possible chops of 12345 are $123 + 45 = 168$ and $1 + 2 + 3 + 4 + 5 = 15$.

(a) If n is divisible by 2, 3, 5 or 7, show that there exists a $2n$-digit integer N with nonzero digits such that no chop of N is divisible by n.
(b) If N is a $2n$-digit integer with nonzero digits, and n is not divisible by 2, 3, 5 or 7, show that there is some chop of N that is divisible by n.

Note: For this problem, please include your proof on separate sheets of paper.

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