Vermont State Mathematics Coalition Talent Search -- March 2025
Test 4 of the 2024-2025 school year

PRINT NAME:_____ Signature:_____
Note: Your signature indicates that answers provided herein are your own work and you have not asked for or received aid in completing this Test.

School_____ Grade _____

Email address:_____

Current Mathematics Teacher: _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets,** in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final. Your solutions may be e-mailed to kmaccormick@cvsdvt.org or be postmarked by **April 3, 2025** and submitted to

Kiran MacCormick
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369 CVU Road
Hinesburg, VT 05461

The Vermont Math Coalition's Talent Search test is prepared by Kiran MacCormick (Math Teacher at Champlain Valley Union HS), Evan Dummit (Associate Teaching Professor at Northeastern University), and Elias Leventhal (Student at Yale University and former talent search winner).

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1. Note that in chess, a bishop attacks all squares that share a diagonal with it. An 8 x 8 chessboard has its squares labeled using the integers 1 through 64 inclusive as shown here.

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

a) What is the minimum number of bishops M required so that every square whose label is a multiple of 3 either has a bishop or is attacked by a bishop?
b) What is the number of different ways of placing M bishops so that every square whose label is a multiple of 3 either has a bishop or is attacked by a bishop?

Answer: a) _____ b) _____

2. This is a relay problem. The answer to each part will be used in the next part.

(a) For an integer n, let $S(n)$ denote the sum of the digits of n and let $D(n)$ denote the total number of positive divisors of n. For
$$n = (10+1)(10^2+1)(10^4+1)...(10^{2^{2024}}+1)-1,$$
compute the value of $D(S(D(D(S(n)))))$.

(b) Let A be the answer to part (a). Suppose x_1 and x_2 are the two real solutions to the equation $10 - 4^{A^x} = 4^{2-A^x}$, with $x_1 > x_2$. Compute $\sqrt{A}^{x_1-x_2}$.

(c) Let B be the answer to part (b). Point P lies inside square $VMTS$ in such a way that the areas of triangles PVM, PMT, and PTS are B, $3B$, and $9B$, respectively. Find the area of triangle PSV.

Answers: (a) _____ (b) _____ (c) _____

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3. Find the number of ordered pairs of nonnegative integers (a, b) such that $9a + 6\sqrt{ab} + b + 2025 = 702\sqrt{a} + 234\sqrt{b}$.

Answer:_____

4. A polynomial $p(x)$ of degree at most 2025 has the property that each of the values $p(0), p(1), \ldots, p(2025)$ is either 0 or 1. Compute the maximum possible value of $p(2026)$.

Answer:_____

5. Triangle ABC has $AB = 13$, $AC = 14$, and $BC = 15$. Squares $ABDE$, $CAFG$, and $BCHI$ are constructed outside triangle ABC, forming a hexagon $DEFGHI$. Squares $DEKJ$, $EFML$, $FGON$, $GHQP$, $HISR$ and $IDUT$ are constructed outside hexagon $DEFGHI$, forming a dodecagon $JKLMNOPQRSTU$. Find the area of dodecagon $JKLMNOPQRSTU$.

Answer:_____

6. Let $g(n) = 2n - 1 - \left\lfloor \left\lfloor n\sqrt{2} \right\rfloor \sqrt{2} \right\rfloor$, where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x. Prove that for all positive integers n, $g(n)^2 + g(n+1)^2 + g(n+2)^2 + g(n+3)^2 + g(n+4)^2$ equals either 1 or 2.

Note: For this problem, please include your proof on separate sheets of paper.

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