Vermont State Mathematics Coalition Talent Search -- November 2025
Test 2 of the 2025-2026 school year

PRINT NAME:_____ Signature:_____

School_____ Grade _____

Email address:_____

Current Mathematics Teacher: _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets,** in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final. Your solutions may be e-mailed to kmaccormick@cvsdvt.org or be postmarked by **December 30, 2025** and submitted to

Kiran MacCormick
Champlain Valley Union High School
369 CVU Road
Hinesburg, VT 05461

Note: The use of AI tools (e.g., ChatGPT) or equivalent technology may not be used to suggest or formulate any specific mathematical steps of a solution. Searching for general mathematical background information is acceptable (e.g., "What does the binomial theorem say?") but you may not ask specific questions about a competition problem. If you are unsure whether a particular use of AI is acceptable, you should not use AI; unauthorized use will result in disqualification.

DISCLAIMER: Please consider completing the following name and image release form in order to help the Vermont State Mathematics Coalition promote our programs and support our pursuit of creating in Vermont significant and lasting improvements in mathematics education and a wider appreciation of mathematics. Signing this release form will not affect the scoring of the competition. If you are under 18, this will need to be signed by a parent or guardian. You only have to complete the image release form once.

NAME AND IMAGE RELEASE FORM

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I have read and understand the above:
Signature _____
Printed name _____
Address _____
Date _____
Signature, parent or guardian _____ (if under age 18)
Parent/guardian name (printed):_____

1. This is a relay problem. The answer to each part will be used in the next part.

   (a) Find the number of ordered triples of positive integers (a, b, c), with the restriction that $c \leq 42$, such that $a^{b^c} = 2025^{2025}$.

   (b) Let A be the answer to part (a). You are playing a game with an infinite set of targets, labeled with the positive integers 1, 2, 3, 4, ..., and the probability that you hit the target labeled n with a given throw is $A/(A+1)^n$. If your throws are independent, what is the probability that if you make two throws, they will hit two different targets?

   (c) Let B be the answer to part (b) and let $N = 1/B$. A degree 4 polynomial $p(x)$ has the property that $p(2) + p(8) = N + 1$, $p(3) + p(7) = 2N - 8$, and $p(4) + p(6) = -12$. Find the value of $p(5)$.

Answers: (a) _____ (b) _____ (c) _____

2. A collection of N lines is drawn in the plane such that each line intersects exactly 2025 other lines. Find the sum of all possible values of N.

Answer:_____

3. Andy, Beth, and Carlos are running laps around a circular track of radius 300 meters. They all run at a constant speed, with Andy taking 10 minutes per lap, Beth taking 12 minutes per lap, and Carlos taking 15 minutes per lap. If they all start at the same point on the track and run clockwise for one hour, find the maximum area attained during this time by the triangle whose vertices are their three locations on the track.

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Answer:_____

4. Find all positive integers n such that $8^n + n + 4$ is divisible by $2^n - n$.

Answer: _____

5. In triangle ABC, we have $\cos A + \cos B + \cos C = \sin A + \sin B + \sin C$ and $\sin A \sin B \sin C = 23/2025$. Find the value of $\tan \dfrac{A}{2} \tan \dfrac{B}{2} \tan \dfrac{C}{2}$.

Answer:_____

6. Define the sequence of positive real numbers $a_0, a_1, a_2 \ldots$ via $a_0 = 1$, $a_1 = 169$, and for $n \geq 1$, $a_{n+1} = 196 a_n + a_{n-1} - 28\sqrt{a_n a_{n-1}}$. Prove that for all $n \geq 0$, a_n is an integer that can be written as the difference of two consecutive perfect cubes (for example, $a_1 = 8^3 - 7^3$).

Note: For this problem, please include your proof on separate sheets of paper.