Vermont State Mathematics Coalition Talent Search -- January 2025
Test 3 of the 2025-2026 school year

PRINT NAME:______________________________________ Signature:________________
Note: Your signature indicates that answers provided herein are your own work and you have not asked for or received aid in completing this Test.

School____ Grade ____________

Email address:____

Current Mathematics Teacher: ____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets,** in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final. Your solutions may be e-mailed to kmaccormick@cvsdvt.org or be postmarked by **March 1, 2026** and submitted to

Kiran MacCormick
Champlain Valley Union High School
369 CVU Road
Hinesburg, VT 05461

Note: The use of AI tools (e.g., ChatGPT) or equivalent technology may not be used to suggest or formulate any specific mathematical steps of a solution. Searching for general mathematical background information is acceptable (e.g., "What does the binomial theorem say?") but you may not ask not specific questions about a competition problem. If you are unsure whether a particular use of AI is acceptable, you should not use AI; unauthorized use will result in disqualification.

The Vermont Math Coalition's Talent Search test is prepared by Kiran MacCormick (Math Teacher at Champlain Valley Union HS), Evan Dummit (Associate Teaching Professor at Northeastern University), and Elias Leventhal (Student at Yale University and former talent search winner).

DISCLAIMER: Please consider completing the following name and image release form in order to help the Vermont State Mathematics Coalition promote our programs and support our pursuit of creating in Vermont significant and lasting improvements in mathematics education and a wider appreciation of mathematics. Signing this release form will not affect the scoring of the competition. If you are under 18, this will need to be signed by a parent or guardian. You only have to complete the image release form once.

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1. How many positive integers divide at least one of the numbers 10^{40}, 20^{30}, 30^{20}, 40^{10}?

Answer:__________

2. This is a relay problem. The answer to each part will be used in the next part.

(a) Evan has a bag of VMTS problems that is 40% algebra, 20% geometry, and 40% counting. Kiran has a bag of VMTS problems that is 20% algebra, 40% geometry, and 40% counting. David has a bag of VMTS problems that is 50% algebra, 40% geometry, and 10% counting. When all three bags are mixed together, the resulting problems are equally distributed among algebra, geometry, and counting. How many times more problems were in Kiran's bag than in David's bag?

(b) Let A be the answer to part (a) and let $N = 12A$. A total of N teams play one another in a round-robin tournament of Extreme Studying. Each pair of teams plays one match and each match has one winner and one loser. If the kth team won w_k total matches and lost l_k total matches for each $1 \leq k \leq N$, what is the maximum possible value of $(w_1^2 + w_2^2 + \cdots + w_N^2) - (l_1^2 + l_2^2 + \cdots + l_N^2)$ among all possible tournament results?

(c) Let B be the answer to part (b). An equilateral triangle has all three of its vertices lie on the graph of the equation $y = x^2$ in the coordinate plane, and one vertex has x-coordinate B. Find the area of the triangle.

Answers: (a) __________________ (b) ____________________ (c) ___________________

3. Goku has a deck of eight cards labeled 1, 2, 3, 4, 5, 6, 7, 8. His current power level is the product of the values on all the cards he has drawn so far. Goku draws cards randomly without replacement from the deck one at a time until the product of the values on his cards exceeds 9,000. Find the expected value of Goku's power level after he stops drawing cards.

Answer:__________

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4. The polynomial $p(x)$ has integer coefficients and is such that $p(1) = 2p(2) = p(3) = 2p(4) = p(5) = 2p(6) = p(7) = 2p(8)$. Find the smallest possible nonzero value of $|p(1)|$.

Answer: ___________

5. A total of thirty identical 4 x 1 tiles and one 1 x 1 tile are arranged to form an 11 x 11 square. Find the number of possible locations where the 1 x 1 square could be placed inside the 11 x 11 square in such a tiling. (Arrangements obtained by rotation and reflection, and the resulting locations of the 1 x 1 square, are considered distinct.)

Answer:___________

6. For a digit k, a positive integer is called k-reversing when multiplying it by k is equivalent to writing its digits in reverse order. For example, 2178 is 4-reversing because $4 \cdot 2178 = 8712$. Determine, with proof, all digits k such that there exist infinitely many k-reversing positive integers.

Note: For this problem, please include your proof on separate sheets of paper.

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