

Vermont State Mathematics Coalition Talent Search -- March 2025

Test 4 of the 2025-2026 school year

PRINT NAME: _____ Signature: _____

Note: Your signature indicates that answers provided herein are your own work and you have not asked for or received aid in completing this Test.

School _____ Grade _____

Email address: _____

Current Mathematics Teacher: _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets**, in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final. Your solutions may be e-mailed to kmaccormick@cvsdvt.org or be postmarked by **April 10th, 2026** and submitted to

Kiran MacCormick
Champlain Valley Union High School
369 CVU Road
Hinesburg, VT 05461

Note: The use of AI tools (e.g., ChatGPT) or equivalent technology may not be used to suggest or formulate any specific mathematical steps of a solution. Searching for general mathematical background information is acceptable (e.g., "What does the binomial theorem say?") but you may not ask not specific questions about a competition problem. If you are unsure whether a particular use of AI is acceptable, you should not use AI; unauthorized use will result in disqualification.

DISCLAIMER: Please consider completing the following name and image release form in order to help the Vermont State Mathematics Coalition promote our programs and support our pursuit of creating in Vermont significant and lasting improvements in mathematics education and a wider appreciation of mathematics. Signing this release form will not affect the scoring of the competition. If you are under 18, this will need to be signed by a parent or guardian. You only have to complete the image release form once.

NAME AND IMAGE RELEASE FORM

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I have read and understand the above:

Signature _____

Printed name _____

Address _____

Date _____

Signature, parent or guardian _____ (if under age 18)

Parent/guardian name (printed): _____

1. Find the number of ordered pairs (m, n) of positive integers with $1 \leq m, n \leq 2026$ such that $3^m + 3^n$ is a perfect square.

Answer: _____

2. Kiran has a fair seven-sided die with faces labeled 1, 2, 3, 4, 5, 6, 7. He rolls the die seven times and scores points based on his rolls: The a th time he rolls the number d , he scores ad points. For example, if Kiran rolls 1, 2, 1, 7, 5, 5, 5, then he scores a total of $1 + 2 + 2 + 7 + 5 + 10 + 15 = 42$ points. Find Kiran's expected score at the end of the game.

Answer: _____

3. This is a relay problem. The answer to each part will be used in the next part.

(a) Kiran starts in the upper left square of the 4×4 grid shown and makes three rightward and three downward moves, each of length 1, to reach the bottom right square. What is the maximum possible sum of the seven squares Kiran occupies on his path, including the starting and ending squares.

8	4	11	1
3	5	6	15
9	13	14	2
16	10	7	12

(b) Let A be the answer to part (a). On each side of an equilateral triangle of side length $\sqrt{3A}$, a rectangle is constructed that shares a side with the triangle and whose interior does not overlap with the triangle. The six vertices of these rectangles that are not vertices of the triangle form a regular hexagon. Find the area of one of these rectangles.

(c) Let B be the answer to part (b) and let $d = B/\sqrt{3}$. Rectangle $ABCD$ has $AB = 8$ and $AC = 5$. Points P on AB , Q on AC , R on CD and S on BD are chosen so that $PQ = QR = RS = SP$ and $QS = \sqrt{d}$. Find $|AQ - QC|$.

Answers: (a) _____ (b) _____ (c) _____

4. Find all ordered pairs of integers (a, b) such that $\frac{a^2 + 20a - 26}{4b^2 - 1} = \frac{a - 2}{2b - 5}$.

Answer: _____

5. A subset S of $\{1, 2, 3, 4, \dots, 2026\}$ is called *progressive* if it contains nine distinct elements in arithmetic progression (for example, such as 1, 3, 5, 7, 9, 11, 13, 15, 17). Prove that more than half of the subsets of $\{1, 2, 3, 4, \dots, 2026\}$ are progressive.

Note: For this problem, please include your proof on separate sheets of paper.

6. Evan and Kiran repeatedly roll a fair 4-sided die labeled with the digits 0, 2, 4, 6. If four consecutive rolls come up 2, 0, 2, 6 (in that order) then Evan wins, while if four consecutive rolls come up 4, 2, 4, 2 (in that order) then Kiran wins. They roll the die until one player wins: so, for example, if the rolls were 6, 2, 2, 4, 2, 6, 2, 0, 2, 6 then Evan wins. What is the probability that Evan wins?

Answer: _____