

Vermont State Mathematics Coalition Talent Search -- October 2026

Test 1 of the 2026-2027 school year

PRINT NAME: _____ Signature: _____

Note: Your signature indicates that answers provided herein are your own work and you have not asked for or received aid in completing this Test.

School _____ Grade _____

Email address: _____

Current Mathematics Teacher: _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets**, in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final. Your solutions may be e-mailed to kmaccormick@cvsdvt.org or be postmarked by **November 6, 2026** and submitted to

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369 CVU Road
Hinesburg, VT 05461

Note: The use of AI tools (e.g., ChatGPT) or equivalent technology may not be used to suggest or formulate any specific mathematical steps of a solution. Searching for general mathematical background information is acceptable (e.g., "What does the binomial theorem say?") but you may not ask not specific questions about a competition problem. If you are unsure whether a particular use of AI is acceptable, you should not use AI; unauthorized use will result in disqualification.

DISCLAIMER: Please consider completing the following name and image release form in order to help the Vermont State Mathematics Coalition promote our programs and support our pursuit of creating in Vermont significant and lasting improvements in mathematics education and a wider appreciation of mathematics. Signing this release form will not affect the scoring of the competition. If you are under 18, this will need to be signed by a parent or guardian. You only have to complete the image release form once.

NAME AND IMAGE RELEASE FORM

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I have read and understand the above:

Signature _____

Printed name _____

Address _____

Date _____

Signature, parent or guardian _____ (if under age 18)

Parent/guardian name (printed): _____

1. In the multiplication below, each letter corresponds to a digit 1-9, with each occurrence of a letter having the same value and with different letters having different values. Find the value of the integer $VMTS$.

$$MS \cdot TST = VMTS$$

Answer: _____

2. This is a relay problem. The answer to each part will be used in the next part.

(a) Evan writes out the infinite repeating decimal expansion of $4/7 = 0.571428571\dots$. He then deletes four digits after the decimal point, obtaining a new number. (For example, he might delete the second, third, fifth and sixth digits, resulting in the number .545714285....) What is the value of the smallest number Evan can obtain this way?

(b) Let A be the answer to part (a) and let p be the numerator of A when written in lowest terms. A mathematics competition has both junior and senior participants. Junior participants know how to solve $p - 2$ different problems, and senior participants know how to solve $p + 2$ different problems. If the number of ways to select an unordered pair of participants is equal to the number of ways to select a single participant along with a problem that participant can solve, find the total number of participants.

(c) Let B be the answer to part (b). Kiran is solving Talent Search problems. You may assume Kiran solves all short-answer problems in the same amount of time, and all proof problems in the same amount of time. It takes Kiran $10B$ minutes to solve five short-answer problems and one proof problem, and it takes Kiran $16B$ minutes to solve one short-answer problem and three proof problems. How many hours would it take Kiran to solve twenty short-answer problems and four proof problems?

Answers: (a) _____ (b) _____ (c) _____

3. Square $ABCD$ has side length 7. Point E is located inside the square $ABCD$ such that $AE = 5$ and such that the circle with center E passing through A also passes through C . If F is the point on the interior of side BC lying on this circle, find the degree measure of angle FAE .

Answer: _____

4. Find the number of positive integers n with $3 \leq n \leq 2026$ for which there exist positive integers $a_1, a_2, \dots, a_n$ with $\frac{1}{a_1^2} + \frac{1}{a_2^2} + \dots + \frac{1}{a_n^2} = 1$.

Answer: _____

5. For a positive integer N , let $s(N)$ denote the sum of the digits of N . Find the number of 2026-digit positive integers N such that $3s(2N) + s(5N) = 2s(N)$. (You may leave your answer in factored form.)

Answer: _____

6. A positive integer n is called *idiphantic* if it has the property that, for any two positive divisors a and b of n , the number $|a + b - \text{lcm}(a, b)|$ is also a divisor of n . Prove that the idiphantic integers consist of 6 along with the prime powers (i.e., the integers of the form p^d for a prime p and a nonnegative integer d).

Note: For this problem, please include your proof on separate sheets of paper.